A Predictive Control Perspective on Electricity Markets

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Outline

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   - Market Volatility and Instability
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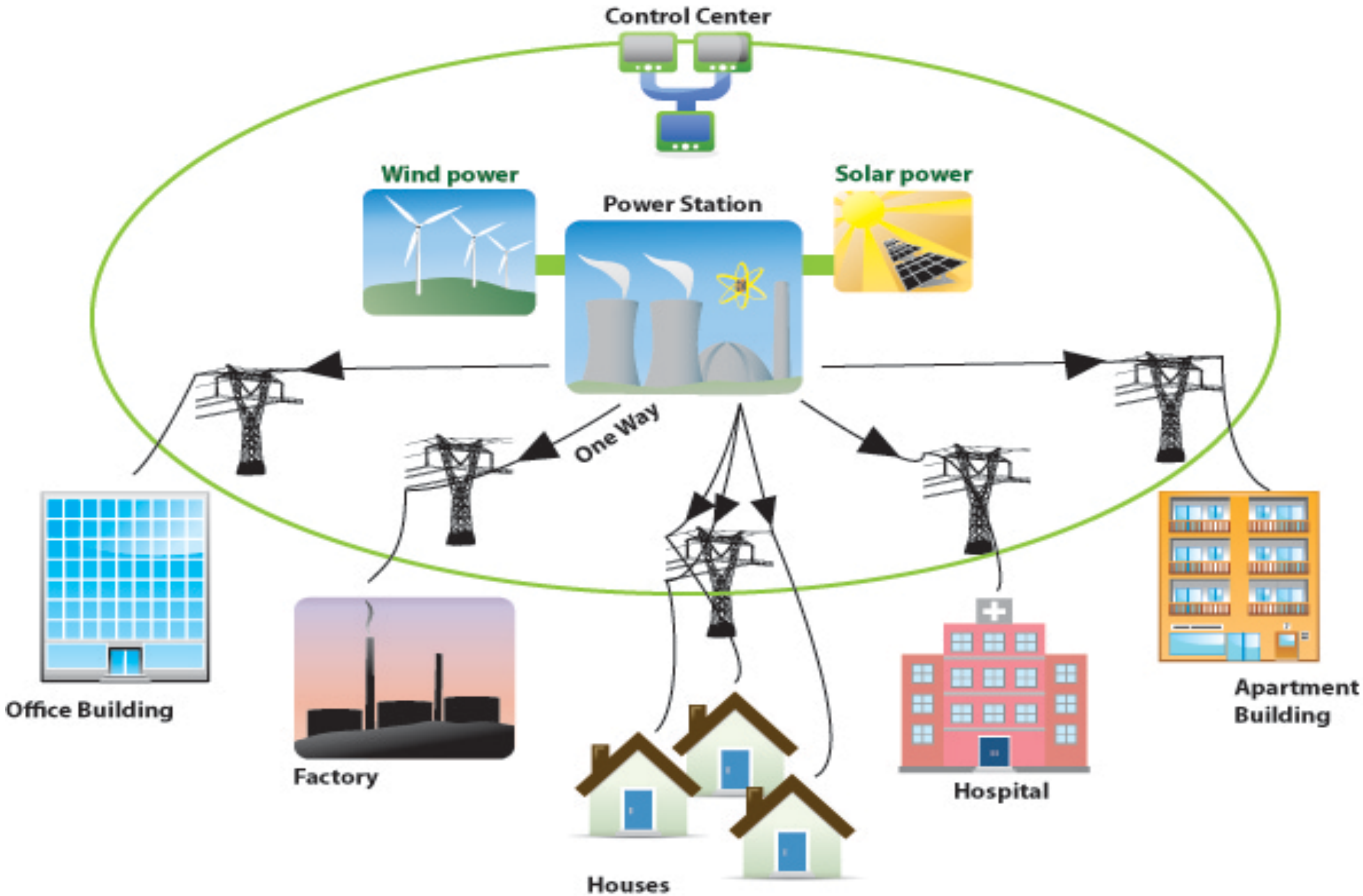
   - Market as Receding Horizon Dynamic Game
3. Stability and Robustness
   - Finite Horizons, Incomplete Gaming, & Forecast Errors
4. Numerical Examples
5. Conclusions and Open Questions

1. Motivation

Current Grid



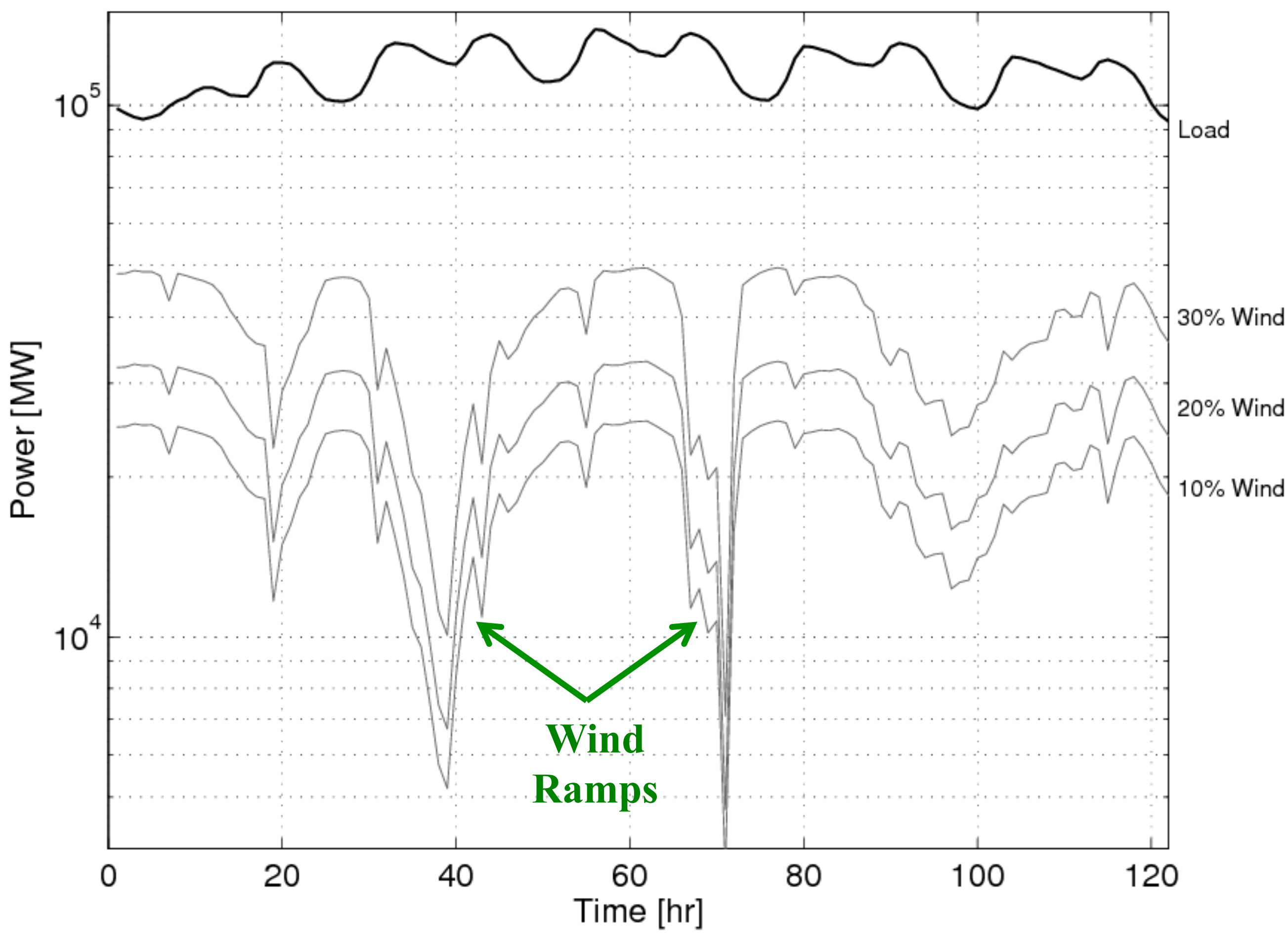
~ 70% Electricity from Central Coal Plants – CO_2 Emissions

Limited Market Control – Demands are Inelastic, No Storage, Slow Generation

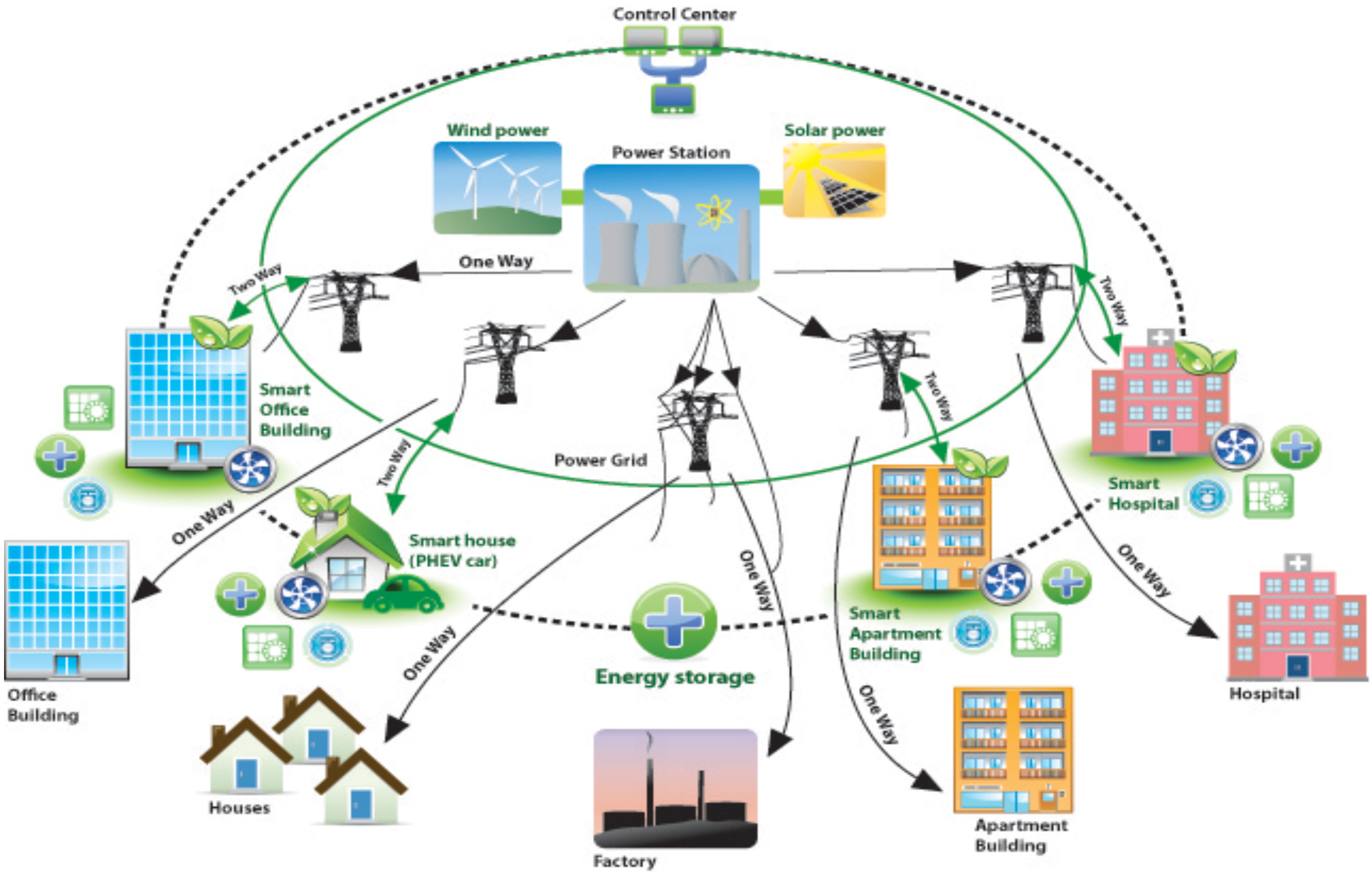
Cannot Sustain High Renewable Supply -Intermittent-

Renewable Supply

Supply -Wind- and Elastic Demands Vary at <u>Higher Frequencies</u>



Next-Generation Grid

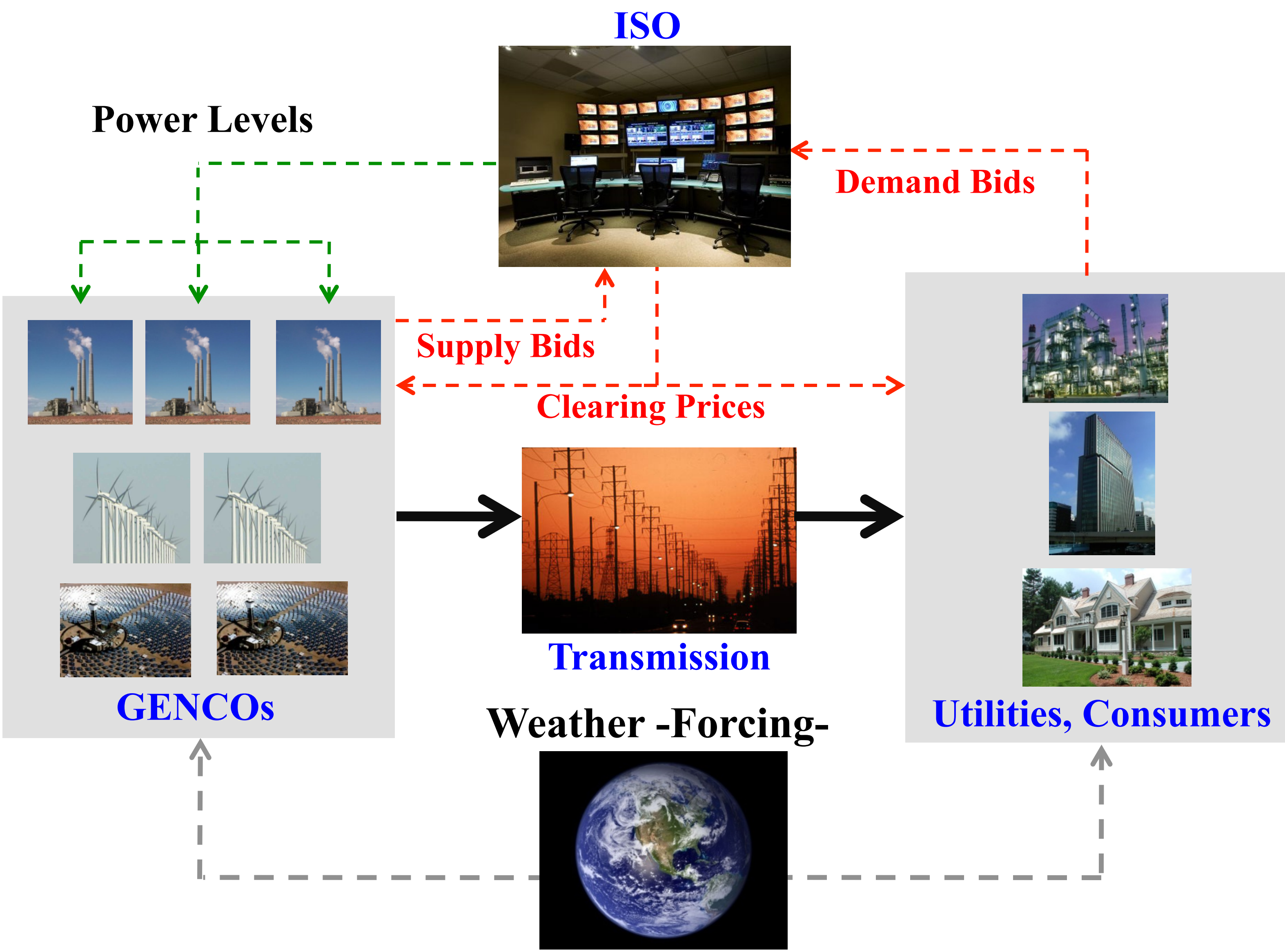


Major Adoption of <u>Renewables</u> -30%-

Real-Time Pricing + Demand Response - <u>Elastic</u> Demands-

Huge Investments in Natural Gas Generation –Faster Response-

Electricity Markets



Dynamic & Uncertain <u>Forcing Factors</u> -Weather- Drive Markets

<u>Volatility</u> Due to Market Friction: (Generation Ramping, Congestion)

State (?) of Energy Markets Modeling

- **Time-series models: (**P. Skantze et al., 2000, Conejo et al. 2005).
- **Agent-based models:** (Veit et al., 2006, Bower and Bunn, 2000).
- **Game-theoretical models (Baldick and Hogan, 2002; Hobbs et al. 2000).**

- **These models tend to be static (assume some for of steady-state behavior e.g. periodicity of the dynamic drivers).**
- **Useful for planning and market design.**
- **But we expect they cannot explain coupled effect of dynamics constraints and nonstationary behavior) of dynamics drivers on future price stability.**

- **Recent, interest in game-theoretical dynamical market models (Mookherjee et al. 2008, de la Torre et al. 2003, ….).**
- **We pursue this direction further looking to enhance stability results.**

Do we have empirical evidence of Instability/ Dynamics

- California prices (*An empirical examination of restructured electricity prices* Knittel, MR Roberts - Energy Economics, 2005).
- Clearly, not driven ONLY by the demand (7/7-7/8) – so preceding state plays a role – points to dynamics.
- Also pointed out by very high correlation at lag 1 (though this by itself is indicative but not confirmatory) .
- If this were a mechanical or electrical system and this were the signal, you would likely argue it was not stable – but this is just a qualitative judgment at the moment.

Figure 4 in Knittel

Figure 2 in Knittel

State (?) of Electricity Market Modeling

- Shares some of the same features in the limit, if the limit makes sense (e.g. is the solution stable in the limit of the small time step while solving linearized problems?) Yes for NLMPC (Zavala and Anitescu SIOPT 2011).

- At the moment, the markets have an hourly clock, so it does not make sense. However, the plan is to increase this frequency, so we will get close.

- We conjecture we will also encounter some of the same issues eventually.

- However, at the moment we focus on what features will this level of modeling uncover.

2. Predictive Control Framework

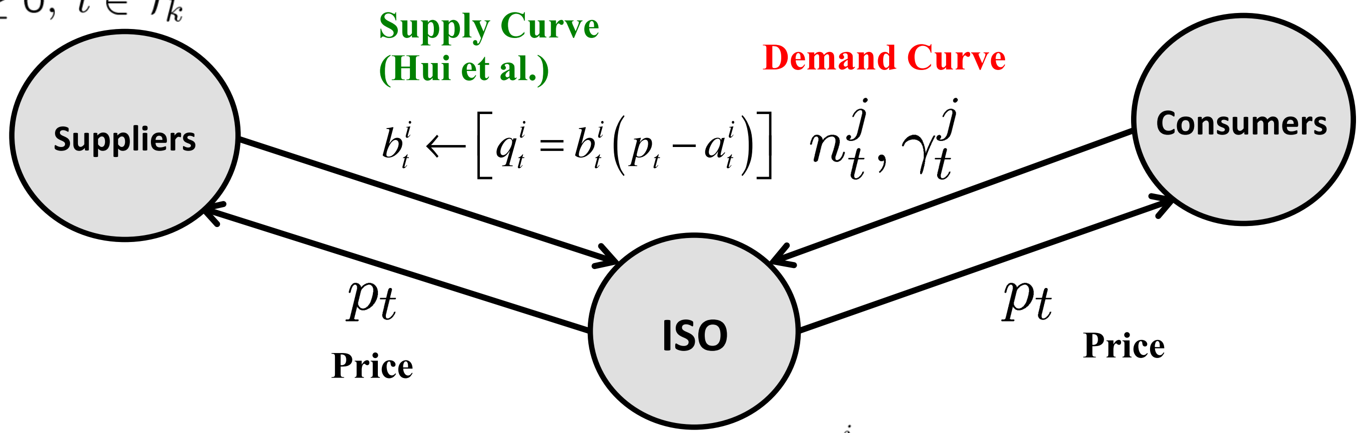
Predictive Control Framework

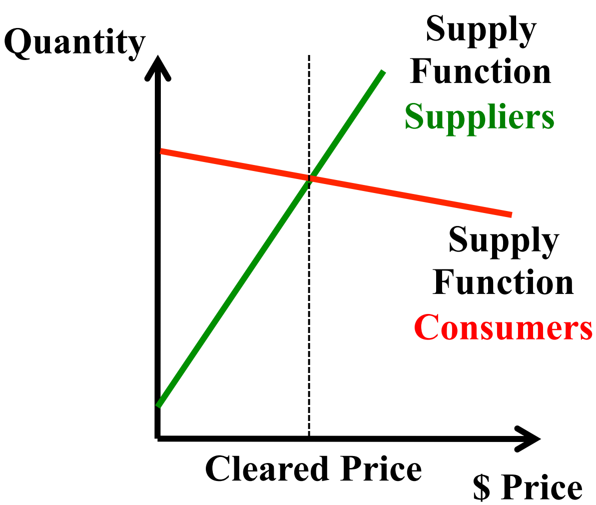
$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} \left(p_t \cdot b_t^i \cdot p_t - c_t^i\left(b_t^i \cdot p_t\right)\right)$$

$$\text{s.t. } \underline{q}^i \leq b_t^i \cdot p_t \leq \overline{q}^i,\ t \in \mathcal{T}_k$$

$$b_t^i \geq 0,\ t \in \mathcal{T}_k$$

$$d_t^j = n_t^j - \gamma_t^j \cdot p_t$$





$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i)\, dq$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i,\ i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_t^j,\ t \in \mathcal{T}_k$$

$$-\underline{r}^i \leq \Delta q_t^i \leq \overline{r}^i,\ i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

$$\underline{q}^i \leq q_t^i \leq \overline{q}^i,\ i \in \mathcal{S}, t \in \mathcal{T}_k$$

$$q_k^i = \text{given},\ i \in \mathcal{S}.$$

Current Markets : Game Runs Incompletely (Jacobi-Like Iteration)

Properties of the involved problems.

Suplier Problem

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} \left(p_t \cdot b_t^i \cdot p_t - c_t^i \left(b_t^i \cdot p_t \right) \right)$$

$$\text{s.t. } \underline{q}^i \leq b_t^i \cdot p_t \leq \overline{q}^i, \; t \in \mathcal{T}_k$$

$$b_t^i \geq 0, \; t \in \mathcal{T}_k$$

$$c_t^i(q_t^i) = h_t^i \cdot q_t^i + \frac{1}{2} g_t^i \cdot (q_t^i)^2.$$

Property 1 If $p_t \geq 0$, $a_t^i = 0$, and $g_t^i \geq 0$, problem (5) is convex. If $p_t > 0$, the problem has a feasible solution for any $\underline{q}^i, \overline{q}^i \geq 0$. If $p_t = 0$, the problem admits a solution only if $\underline{q}_t^i = 0$.

ISO Problem

$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i, \; i \in \mathcal{S}, t \in \mathcal{T}_k^{-}$$

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_t^j, \; t \in \mathcal{T}_k$$

$$-\underline{r}^i \leq \Delta q_t^i \leq \overline{r}^i, \; i \in \mathcal{S}, t \in \mathcal{T}_k^{-}$$

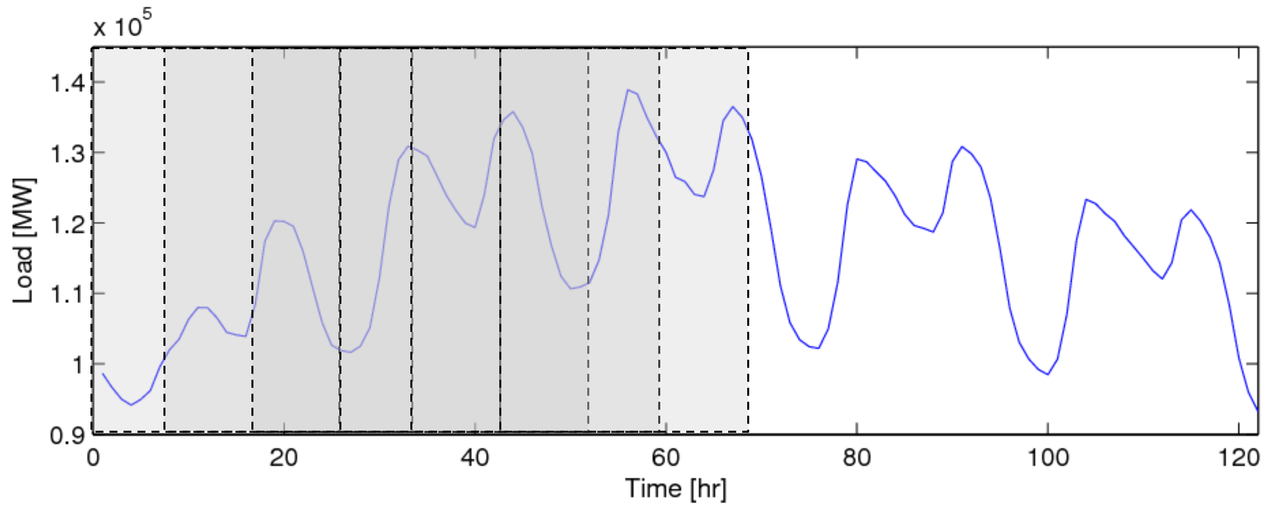
$$\underline{q}^i \leq q_t^i \leq \overline{q}^i, \; i \in \mathcal{S}, t \in \mathcal{T}_k$$

$$q_k^i = \text{given}, \; i \in \mathcal{S}.$$

Property 2 If $b_t^i \geq 0$, problem (8) is convex. The problem has a feasible solution if $\sum_{i \in \mathcal{S}} \underline{q}^i \leq \sum_{j \in \mathcal{C}} d_j^t \leq \sum_{i \in \mathcal{S}} \overline{q}^i$ holds. If $b_t^i > 0$, feasibility holds for any $\underline{q}_t^i, \overline{q}_t^i \geq 0$. If $b_t^i = 0$, the problem admits a solution only if $\underline{q}_t^i = 0$.

Predictive Control Framework

Current Markets: **Game Implemented Over Receding Horizon – Load**



At k solve over $\mathcal{T}_k = \{k, ..., k+T\} \Rightarrow$ Implement Price p_k

At $k+1$ solve over $\mathcal{T}_{k+1} = \{k+1, ..., k+1+T\} \Rightarrow$ Implement Price p_{k+1}

Key Issues:

- **How to Measure Dynamic Market Stability?**
- **Stability Conditions Under Finite Horizon**
- **Stability Conditions Under Incomplete Gaming**
- **Robustness Bounds**
- **Effect of Market Design: Frequency, Horizon, Strategic, Stabilizing Constraints**
- **Effect of Mechanistic Effects: Ramps, Topology, Congestion**

Dynamic Optimization and DVI.

- **Connection to DVI – it is a dynamic (sequential? Recursive?) discrete optimization/VI problem**

$$
\begin{array}{ccccccc}
\to x^k \to & \min_x F(x,x^k,u^k) & \to x^{k+1} \to & \min_x F(x,x^{k+1},u^{k+1}) \to \ldots. \\
 & \uparrow & & \uparrow \\
 & u^k & & u^{k+1}
\end{array}
$$

$$
\begin{array}{ccccccc}
\to x^k \to & x \in SOL(F(\bullet,x^k,u^k),K) & \to x^{k+1} \to & x \in SOL(F(\bullet,x^{k+1},u^{k+1}),K)\ldots. \\
 & \uparrow & & \uparrow \\
 & u^k & & u^{k+1}
\end{array}
$$

- **It has the same structure as time-stepping for DVI**

$$
\begin{array}{rcl}
y^{h,(i+1)} & = & y^{h,i} + h\tilde{f}\left(\tilde{t}^{h,(i+1)}, \theta_1 y^{h,i} + (1-\theta_1)y^{h,(i+1)}, x^{h,(i+1)}\right) \\
x^{h,(i+1)} & \in & SOL(K; \tilde{F}(\tilde{t}^{h,(i+1)}, \theta_2 y^{h,i} + (1-\theta_2)y^{h,(i+1)}, \cdot)) \\
y(0) & = & y_0.
\end{array}
$$

Dynamic Optimization and DVI.

- Shares some of the same features in the limit, if the limit makes sense (e.g. is the solution stable in the limit of the small time step while solving linearized problems?) Yes for NLMPC (Zavala and Anitescu SIOPT 2011).
- At the moment, the markets have an hourly clock, so it does not make sense. However, the plan is to increase this frequency, so we will get close.
- We conjecture we will also encounter some of the same issues eventually.
- However, at the moment we focus on what features will this level of modeling uncover.

3. Stability and Robustness

Market Stability (A Proposal)

Constrained Market Clearing

$$
\begin{aligned}
\min_{q_t^i, \Delta q_t^i} \ & \sum_{t\in\mathcal{T}_k} \varphi_t := \sum_{t\in\mathcal{T}_k}\sum_{i\in\mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i)dq \\
\text{s.t. } & q_{t+1}^i = q_t^i + \Delta q_t^i, \ i\in\mathcal{S}, t\in\mathcal{T}_k^- \\
& \sum_{i\in\mathcal{S}} q_t^i \geq \sum_{j\in\mathcal{C}} d_t^j, \ t\in\mathcal{T}_k \qquad (p_t) \\
& -\underline{r}^i \leq \Delta q_t^i \leq \overline{r}^i, \ i\in\mathcal{S}, t\in\mathcal{T}_k^- \\
& \underline{q}^i \leq q_t^i \leq \overline{q}^i, \ i\in\mathcal{S}, t\in\mathcal{T}_k \\
& q_k^i = \text{given}, \ i\in\mathcal{S}.
\end{aligned}
$$

Unconstrained Market Clearing (Utopia)

$$
\begin{aligned}
\min_{q_t^i} \ & \sum_{t\in\mathcal{T}_k} \varphi_t = \sum_{t\in\mathcal{T}_k}\sum_{i\in\mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i)dq \\
\text{s.t. } & \sum_{i\in\mathcal{S}} q_t^i \geq \sum_{j\in\mathcal{C}} d_t^j, \ t\in\mathcal{T}_k \qquad (\bar{p}_t) \\
& \underline{q}^i \leq q_t^i \leq \overline{q}^i, \ i\in\mathcal{S}, t\in\mathcal{T}_k,
\end{aligned}
$$

$$\bar{\varphi}_t$$

Property: For Fixed b_t^i **,** $\bar{\varphi}_t \leq \varphi_t, \forall t \in \mathcal{T}_k$

Definition: Market Efficiency. $\eta_t = \dfrac{\bar{\varphi}_t}{\varphi_t} \in [0,1]$

Definition: Market Stability. The market given by the ISO/Supplier/Consumer game is stable if, given $\eta_0 \in \{\eta \,|\, \eta \geq \epsilon\}$ **we have generation and demand sequences such that** $\eta_t \in \{\eta \,|\, \eta \geq \epsilon\}, \ \forall t.$

Lyapunov Stability

Lyapunov Function = **<u>Indicator Function</u>** **(Sufficient Conditions, Compare Designs)**

Definition: Market Summarizing State.

$$\delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t \text{ with } \alpha(\eta, \epsilon) \leq 1 \text{ iff } \eta \leq \epsilon.$$

Observations: - **Market Stability Implies Stability of Origin for Summarizing State**
- **Maximizing Efficiency Implies Minimizing Summarizing State**

Abstract ISO (AISO) Clearing Problem:

$$\begin{aligned}
\min_{u_{\mathcal{T}_k^-}} & \sum_{t \in \mathcal{T}_k{}^-} (\delta_{t+1} - \delta_t) \\
\text{s.t. } & u_{\mathcal{T}_k} \in \Omega(\delta_k, d_{\mathcal{T}_k}) \\
& \delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t, \ t \in \mathcal{T}_k{}^- \\
& \eta_t \geq \epsilon, \ t \in \mathcal{T}_k \longleftarrow \\
& \delta_k = \text{given.}
\end{aligned}$$

Extra stabilizing Constraint

Candidate Lyapunov Function.

$$V_T(\delta_k, d_{\mathcal{T}_k}) := -\sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) = \delta_k - \delta_{k+T}.$$

Lyapunov Stability

Infinite Horizon: If game with horizon $T = \infty$ is feasible for AISO then, the market is stable.

Proof:

$$\begin{aligned}\Delta V_T(\delta_k) &= V_\infty(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_\infty(\delta_k, m_{\mathcal{T}_k}) \\ &= \sum_{t=k+1}^{\infty} (\delta_t^{k+1} - \delta_{t+1}^{k+1}) - \sum_{t=k}^{\infty} (\delta_t^k - \delta_{t+1}^k) \\ &= \left(\delta_{k+1} - \delta_\infty^{k+1}\right) - \left(\delta_k - \delta_\infty^k\right) \\ &= -\left(\delta_k - \delta_{k+1}\right) \\ &= \boxed{(\alpha(\eta_{k+1}, \epsilon) - 1)} \cdot \delta_k \\ &\leq 0\end{aligned}$$

Accumulation Term, negative by enforcing proposal

Finite Horizon: Define Terminal Cost:

$$\Xi_k^1 := |V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_{T-1}(\delta_{k+1}, m_{\mathcal{T}_k})|, \; \Xi_k^1 \to 0, \; T \to \infty$$

Finite Horizon: If game with horizon $T < \infty$ is feasible and the terminal cost is bounded by accumulation term, then the market is stable.

Proof:

$$\begin{aligned}\Delta V_T(\delta_k) &= V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_k, m_{\mathcal{T}_k}) \\ &= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k + \Xi_k^1 \\ &\leq 0\end{aligned}$$

- **Price Volatility Related to Ramp Limits** $\|p_t - \bar{p}_t\| \leq L(\|\bar{r} - \bar{q}\| + \|\underline{r} - \underline{q}\|)$

-Key Outcome: - Incomplete Gaming Cannot be Guaranteed to be Stable

- Stabilizing Constraint "Filters Out" Spurious Bids

Stability of the Game Formulation – A Lemma

Lemma 1 *Consider the following block matrix (where the diagonal blocks are square).*

$$L = \begin{bmatrix} G & 0 & 0 & A & 0 \\ I & P & 0 & BH & 0 \\ 0 & -B^{-2} & B^{-1} & M^T & N^T \\ 0 & 0 & M & 0 & 0 \\ 0 & 0 & N & 0 & 0 \end{bmatrix}$$

We make the following assumptions:

[A1] The matrix G is invertible.
[A2] The matrices P, B are diagonal and positive, with entries equal to the prices p_t and bidding parameters b_i^t, respectively.
[A3] The blocks G, P, B have the same dimensions.
[A4] The matrix $[M^T N^T]$ has full column rank.
[A5] The diagonal entries of B are bounded below.

Then, there exist positive values q_*, p_*, independent of G such that, if $p_t \cdot b_t^i > q_*$ and $p_t > p_*$, then the matrix L is nonsingular.

Stability of the game formulation: Results

Theorem 1 *Let J be the reduced Jacobian of the game (5) and (11) (the Jacobian of the coupled KKT conditions of the game, with the variables that reached their bounds eliminated). Then, if at a solution of the game each of the optimization problems satisfies LICQ and the prices p_t, $t \in \mathcal{T}$ and the production q_t^i, $t \in \mathcal{T}$, $i \in \mathcal{S}$ values are large enough, then J is invertible.*

- **If we have strict complementarity, this is sufficient to show stability**

Theorem 2 *Assume that a solution of the unconstrained game $\bar{\varphi}_t, \bar{p}_t, \bar{\eta} = 1, t \in \mathcal{T}$ given by (7) and (10) is locally stable. Then, there exist Lipschitz constants $L_\varphi, L_p, L_\eta \geq 0$ such that the solution of the constrained game φ_t, p_t, η_t, $t \in \mathcal{T}$ given by (7) and (11) satisfies,*

$$
\begin{aligned}
|\varphi_t - \bar{\varphi}_t| &\leq L_\varphi \sum_{i \in \mathcal{S}} \left(|\overline{r}^i - (\overline{q}^i - \underline{q}^i)| + |\underline{r}^i - (\overline{q}^i - \underline{q}^i)| \right) \\
|p_t - \bar{p}_t| &\leq L_p \sum_{i \in \mathcal{S}} \left(|\overline{r}^i - (\overline{q}^i - \underline{q}^i)| + |\underline{r}^i - (\overline{q}^i - \underline{q}^i)| \right) \\
|\eta_t - 1| &\leq L_\eta L_\varphi \sum_{i \in \mathcal{S}} \left(|\overline{r}^i - (\overline{q}^i - \underline{q}^i)| + |\underline{r}^i - (\overline{q}^i - \underline{q}^i)| \right).
\end{aligned}
$$

- **Therefore our proposal "stabilizing" program will be feasible if ramp rates are large enough (and this includes a non trivial range).**

Robustness

Effect of Forecast Errors

Define Cost Perturbation:

Predicted State with Forecast **State with True Data**

$$\Xi_k^2 := |V_T(\bar{\delta}_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}})|.$$

Key: Boundedness of Perturbation Requires Game –Numerical- Stability.

Numerical Stability: If at a solution of the game the players problems satisfy LICQ and the <u>clearing prices are sufficiently large</u>, the solution is stable and Lipschitz continuous on the data.

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} \left(p_t \cdot b_t^i \cdot p_t - c_t^i \left(b_t^i \cdot p_t\right)\right)$$
$$\text{s.t. } \underline{q}^i \leq b_t^i \cdot p_t \leq \overline{q}^i, \; t \in \mathcal{T}_k$$
$$b_t^i \geq 0, \; t \in \mathcal{T}_k$$

$p_t \rightarrow 0$ **Destroys Curvature (Excess Supply)**

Robust Finite Horizon: If game with horizon $T < \infty$ is feasible and the terminal cost and cost perturbation are bounded by accumulation term, then the market is stable. Similar result for incompletely converged game.

Proof:

$$\begin{aligned} \Delta V_T(\delta_k) &= V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_k, m_{\mathcal{T}_k}) \\ &= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k + \Xi_k^1 + \Xi_k^2 \\ &\leq 0 \end{aligned}$$

4. Numerical Examples

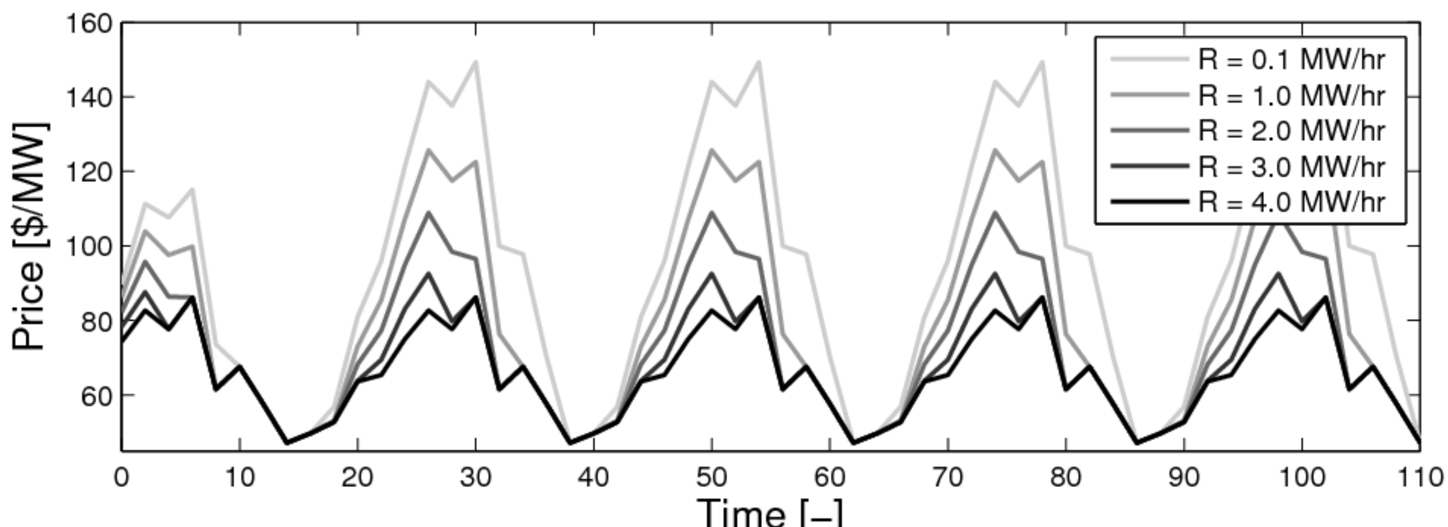
Dynamic Electricity Markets

Supply Function-Based Dynamic Game Models *Kannan & Zavala., 2010*

- Linear Complementarity Problem

$$
\left.
\begin{array}{l}
\displaystyle \max_{a_i^t,b_i^t,q_i^t} \quad \sum_{t=1}^{T} \Bigg(\overbrace{\left(\frac{q_i^t + a_i^t}{b_i^t}\right)}^{\text{Price}} q_i(t) - C_i(q_i(t)) \Bigg) \\
\left\{
\begin{array}{ll}
 & q_i^t \le cap_i^t \\
 & q_i^{t+1} - q_i^t \le R_i^t \\
s.t. & \dfrac{q_i^t + a_i^t}{b_i^t} = \dfrac{c^t + \sum_{i=1}^N a_i^t}{d^t + \sum_{i=1}^N b_i^t} \\
 & q_i^t \ge 0
\end{array}
\right\} , \forall t = 1, 2, .., T \quad \underset{\text{Horizon}}{}
\end{array}
\right\} \forall i = 1, ..., P \quad \underset{\text{Players}}{}
$$

Effect of Ramp Constraints on Dynamic Equilibria



Effects of Incomplete Gaming

- **In reality, the gaming is not complete (e.g. the suppliers never use the converged prices in their bids).**
- **What is the effect of incomplete gaming on stability?**
- **Problem we would like to solve**

$$\min_{b_t \in K_b} f^b(b_t, q_t) \Leftrightarrow \min_{q_t \in K_q} f^q(b_t, q_t)$$

- **Incomplete solution:**

$$b_t^{(k)} = \arg\min_{b_t \in K_b} f^b(b_t, q_t^{(k-1)}) \Rightarrow q_t^{(k)} = \min_{q_t \in K_q} f^q(b_t^{(k)}, q_t)$$

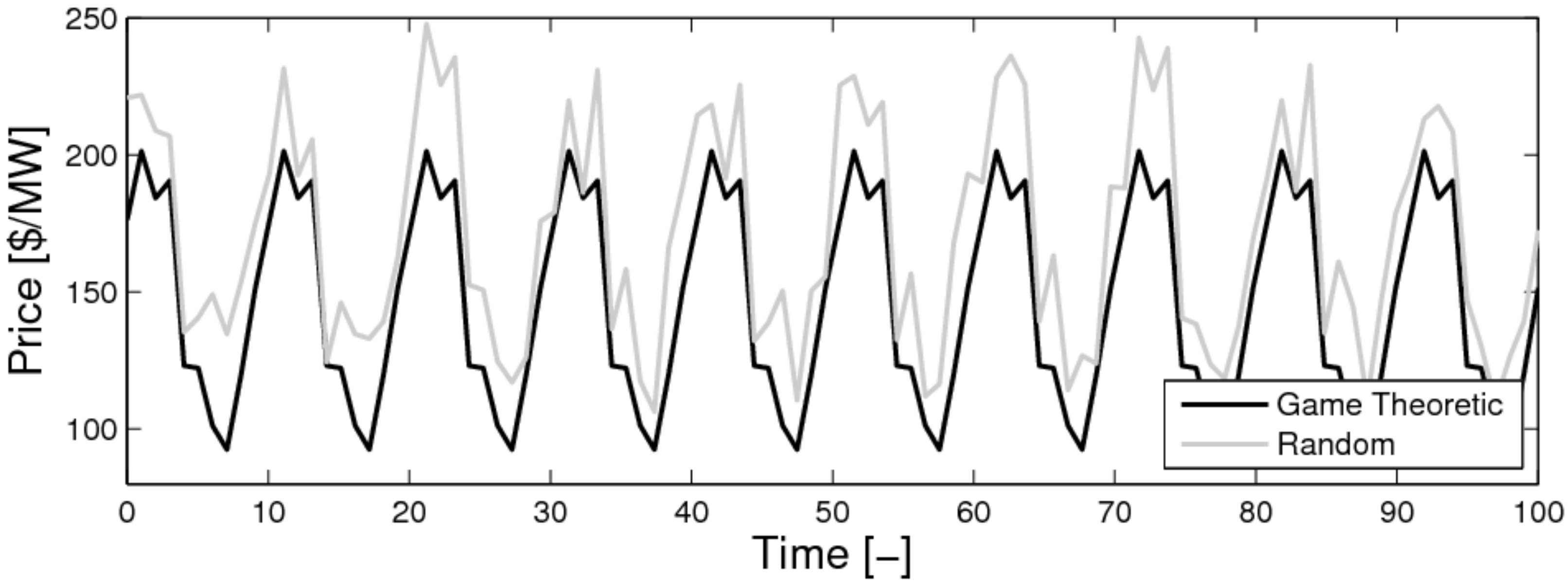
- **Some references argue that current way market works is one or a few such (GS) iterations.**

Dynamic Electricity Markets

Non-Gaming Behavior

Some Players -Intentionally or Unintentionally- Bid Suboptimally

Introduces Noise in Equilibrium – Can be Inferred from Data



Huge Potential for Dynamic Market Models

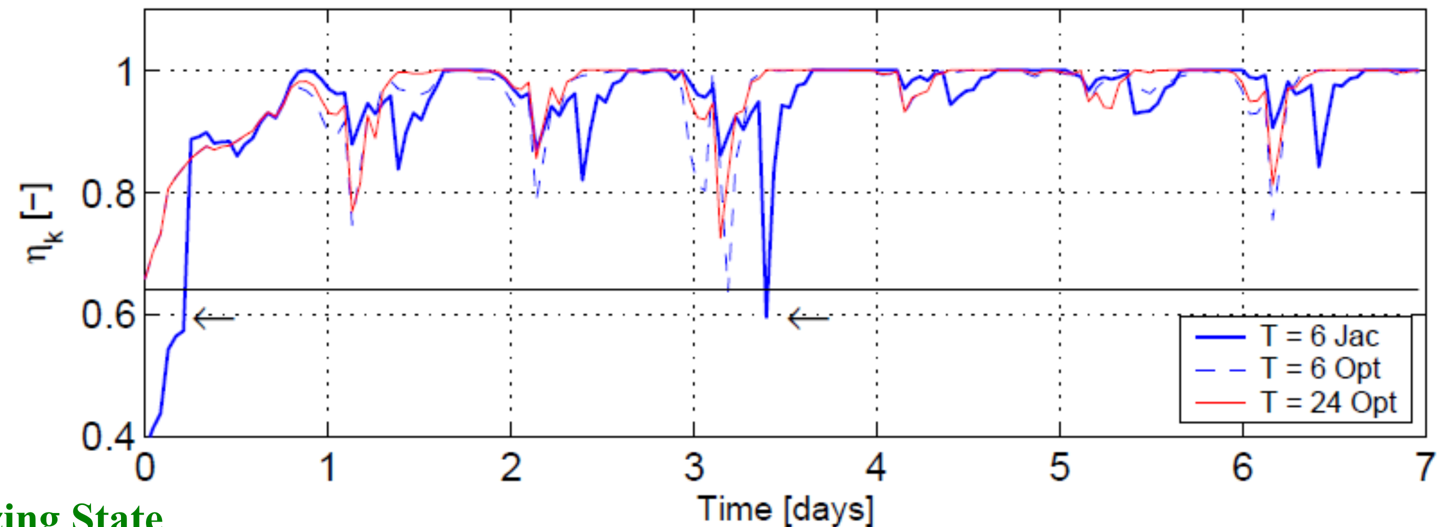
- **Mechanistic Price Forecasting, Market Design and Monitoring**
- **Data Assimilation and State Estimation**

Stability

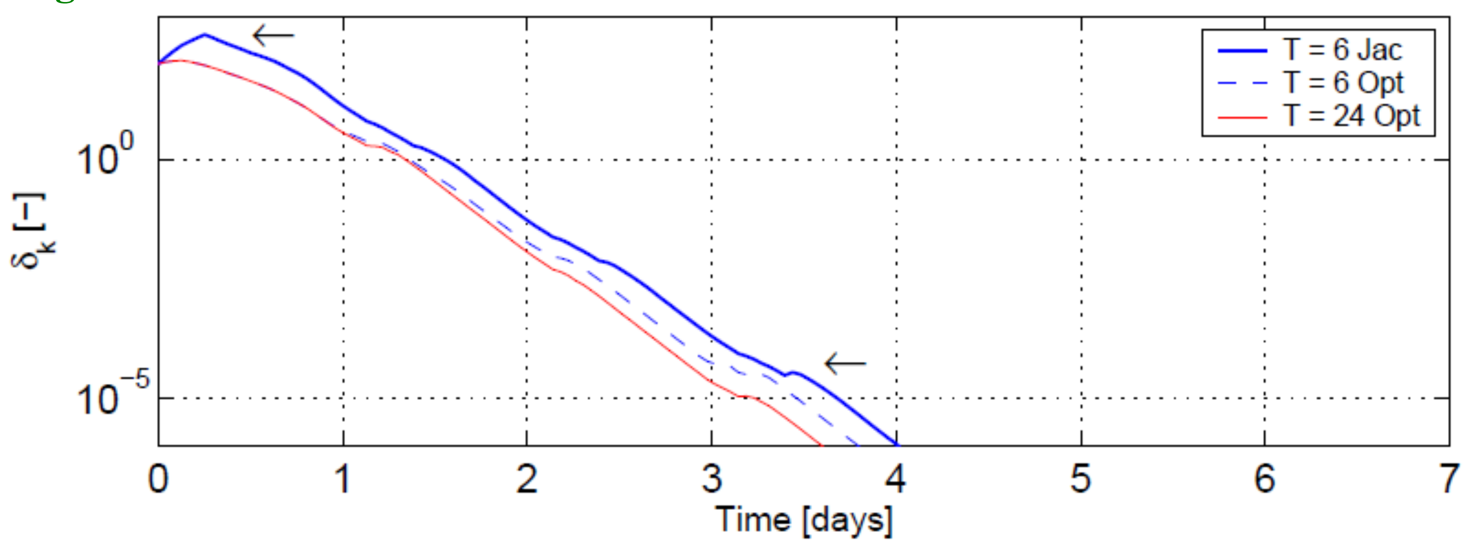
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming
- 6 Hours Horizon, Complete Gaming
- 24 Hours Horizon, Complete Gaming

Efficiency



Summarizing State

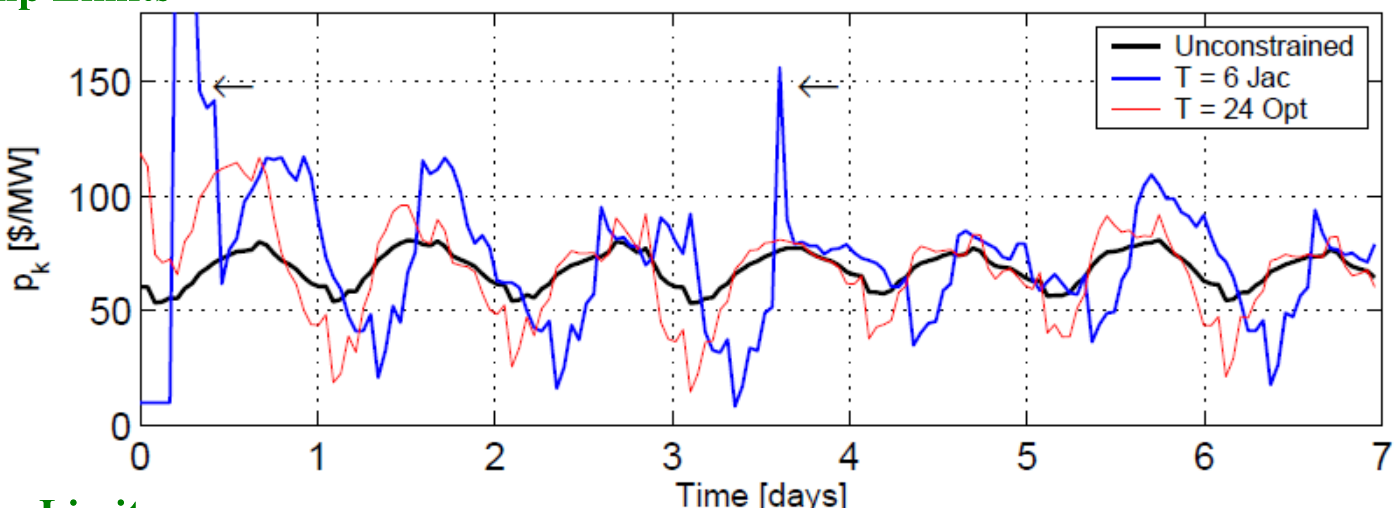


Stability

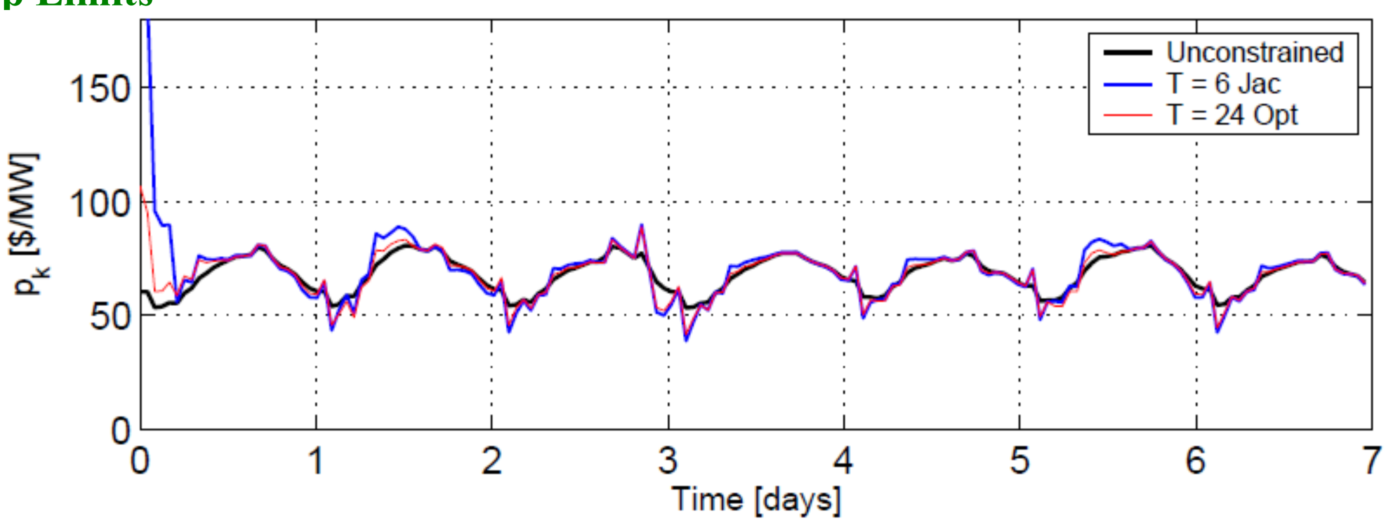
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming
- 6 Hours Horizon, Complete Gaming
- 24 Hours Horizon, Complete Gaming

Tight Ramp Limits



Lose Ramp Limits



5. Conclusions and Open Questions

Conclusions and Open Questions

Predictive Control Provides a Framework for Market Analysis

- Advantage: Captures Mechanistic and Physical Effects
- Advantage: Captures Decision-Making Rationale (Receding Horizon)
- Issue: Market Inherently Dynamic (No Natural Equilibrium)
- Issue: Market Stability and Efficiency Definitions are Subjective

Potential Extensions:

- Day-Ahead and Real-Time Markets
- Stochastic Formulations (Effects of Risk on Stability)
- Distributed Optimization Algorithms
- Continuous-Time (Closer to Physical Domain)
- Alternative Designs (Stabilizing Constraints)

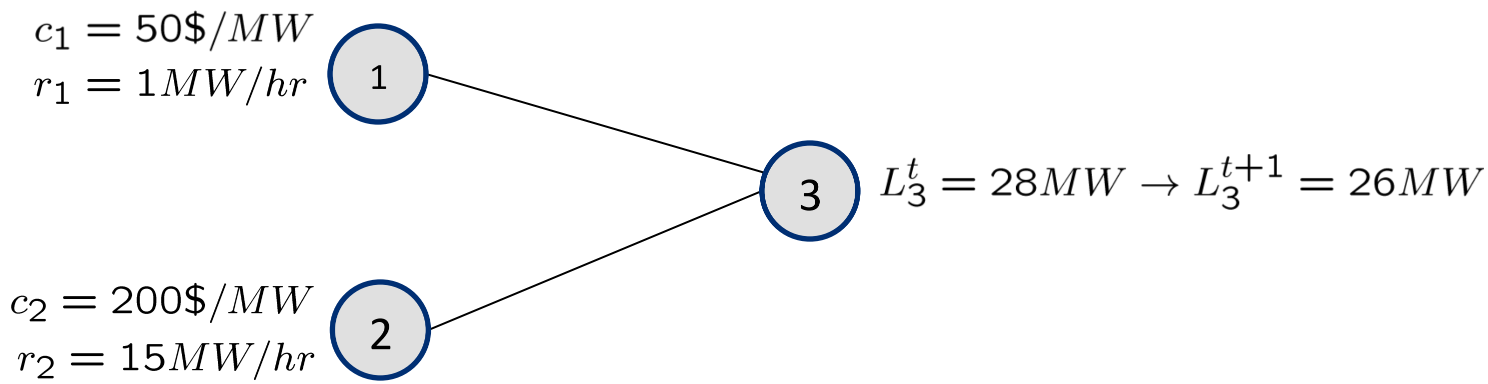
Alternative Frameworks: Stochastic Stability

Alternative Definitions: Economic Efficiency and Price Volatility

Theoretical Analysis:

- How do we choose epsilon?
- Extend stability beyond strict complementarity
- Differential Variational Inequalities?

Market Instability and Ramp Constraints



No Ramp Constraints

$$\lambda^t = 50\$/MW(28,0) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 27MW$

$G_{t-1}^2 = 1MW$

$$\lambda^t = 50\$/MW(28,0) \rightarrow \lambda^{t+1} = 0\$/MW(27,0)$$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 26MW$

$G_{t-1}^2 = 2MW$

$$\lambda^t = 50\$/MW(27,1) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramp Constraints (with Foresight)

$G_{t-1}^1 = 27MW$

$G_{t-1}^2 = 1MW$

$$\lambda^t = 55.35\$/MW(27,1) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramps Lead to Market Volatility – Propagation Through Initial Conditions (Need Foresight)

A Predictive Control Perspective on Electricity Markets

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UIUC Seminar
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